Course Learning Outcomes for Unit III

Upon completion of this unit, students should be able to:

3. Explain Newton's laws of motion at work in common phenomena.
   3.1 Illustrate the relation of the law of universal gravitation to Newton's second law.
   3.2 Distinguish between gravitational acceleration (*g*) and gravitational constant (*G*).
   3.3 Evaluate gravitational field strength when mass and radius of an object are given.

5. Identify the total mechanical energy conservation.
   5.1 Explain total mechanical energy conservation for orbital motion.
   5.2 Describe escape velocity when gravitational potential energy is balanced with kinetic energy.

Required Unit Resources

Chapter 5: Dynamics of Uniform Circular Motion

Chapter 6: Work and Energy

Unit Lesson

Circular Motion

Consider an object moving in a circular orbit of the radius (r) about a center of force. From symmetry, the speed (v) of the body must be constant, but the direction of the velocity vector is constantly changing. Such a changing velocity represents an acceleration ($a = v^2 / r$), which is the centripetal acceleration that maintains the circular orbit. In this circular motion, Newton's third law plays an important role.

A moving object has a tendency to keep moving with constant speed according to Newton's first law. What is the driving force of this system? It is the centripetal (center-seeking) force to keep up the continuous circular motion. The direction of the centripetal force is inward. This is the action force. The reaction force is called the centrifugal (center-fleeing) force. The direction of the centrifugal force is outward. It is a fictitious force. If the object is released, that is, absence of the centripetal force, from the circular motion, it will fly out not because of the centrifugal force, but because of Newton's first law, the law of inertia. Unless there is no external force, an object that is at rest stays at rest and one that is in motion continues to move.

Projectile Motion

When an object moves with a curved path near the Earth's surface under the influence of gravity, its motion is called projectile motion as we have learned in the previous unit. Recall Example 6 from section 3.3 in Chapter 3 of your eTextbook (Cutnell et al., 2022).

If we ignore air resistance, the horizontal motion of the projectile does not slow down; its velocity is constant. In other words, the horizontal component of the acceleration is zero. However, the vertical component of the velocity is not constant. In addition, the vertical component of the acceleration is downward acceleration, gravitational acceleration (g).

Weightlessness and Free Fall

Suppose you are in an elevator. If the elevator is not accelerating, your weight (W) is just your mass (m) times the gravitational acceleration (g). In fact, two forces are acting on you; the weight (W) and the normal force (F).

According to Newton's second law, in the vertical direction, $ma = F - W = F - mg$. That is, normal force $F = m(g + a)$. Here, g is positive, but it may be either positive for upward acceleration or negative for the downward acceleration of the elevator. If the elevator is in an upward motion, apparent weight (or normal force) is greater than your true weight. On the other hand, if the elevator is in a downward motion, the apparent weight is smaller than your true weight. In a special case, when the acceleration is equal to g, that is, $a = -g$, or free fall, the apparent weight becomes zero: weightlessness. The same phenomenon occurs when an object is circling around the Earth. The orbiting satellite, which accelerates toward the center of the Earth, is also in free fall. Please review Figure 5.17 in section 5.6 of Chapter 5 in your eTextbook (Cutnell et al., 2022).

Over a long period of time, weightlessness is harmful to humans, thus, a rotating space station in a wheel shape can be created to create artificial gravity. It is balanced with the centripetal force, mv^2 / r, of the system. That is $mg = mv^2 / r$. Here, m is the mass of an astronaut, r is the distance from the axis to the surface of the station, and v is the rotating speed. See Figure 5.19 in section 5.6 of Chapter 5 in your eTextbook (Cutnell et al., 2022).

Sample Question 1: If the mass of an astronaut is 80 kg and the radius of the rotating space station is 1 km, what is the rotation speed v?

Solution: The gravitational force is balanced with the centripetal force; $mg = mv^2 / r$. So, $v = (rg)^{1/2}$ = 100 m/s.
The rotating speed does not affect the mass of the astronaut.

Newton's Law of Universal Gravitation

Newton speculated about the highest reachable point by the force of gravity on the Earth. He realized that there is a limit and concluded that the orbital motion of the moon around the Earth is maintained by the gravitational force (Hewitt, 2015). Suppose you throw a stone horizontally from a high place. The stone falls to the ground because of gravity. However, if you throw the stone with great speed, it will move farther and farther away from where you are standing before falling to the ground. When the speed is great enough, the stone will eventually circle around the Earth. This is the projectile motion where the projectile falls in the gravitational field but never touches the ground. This logical consideration can be applied to explain the orbital motion of the moon. Newton concluded that the moon is falling in its pathway around the Earth because of the gravitational acceleration.

Newton extended the above idea to any two objects in the universe in order to explain the interaction between them. Newton's law of universal gravitation postulates that there is an attractive force between the two objects (Hewitt, 2015). The force between two objects in the universe is proportional to the product of two masses m and M and is inversely proportional to the square of distance r between two objects, $F = GmM / r^2$, where G = (6.67×10^{-11} N m^2/kg^2) is the universal gravitational constant. This is the case when the gravitational acceleration (a) is equal to g in the second law of Newton; $a = g$, and thus, $g = GM / r^2$. The constant, G was measured by Henry Cavendish 100 years after Newton announced his theory. It was not an easy task because of the extremely small value of gravitation attraction.

Sample Question 2: What is the magnitude of the gravitational force between the sun and the Earth? The distance between the sun and the Earth is 1AU = 1.50×10^{11} m. The mass of the Earth is $m = 5.98 \times 10^{24}$ kg, and the mass of the Sun is $M = 1.99 \times 10^{30}$ kg.

Solution: From $F = GmM / r^2 = 6.67 \times 10^{-11} \text{ x } 5.98 \times 10^{24} \text{ x } 1.99 \times 10^{30} / (1.50 \text{ x } 10^{11})^2 = 3.5 \times 10^{22}$ N

Kepler's Three Empirical Laws for Planetary Motion

Johannes Kepler (1571–1630) was a German astronomer and had endless enthusiasm for researching the solar system. It took him more than 20 years to realize through his calculations the exact shape of the planets' orbitals. He tested many different kinds of models using his teacher, Tyco Brache's enormous data set. Brache had accumulated very exact planetary observations without even the use of telescopes. Kepler established three important empirical laws of planetary motion, the law of elliptical orbit, the law of areas, and the law of the relation between period and distance, to describe and understand the motion of the solar system.

The mechanical motion of our solar system obeys gravitational law, and planets are orbiting around the sun whose mass is heaviest. The orbital shape is not circular but elliptical. Some comets have parabolic or hyperbolic orbits. These well-known mechanics were not easily obtained. Since ancient times, the sky was considered a realm of gods, so perfectness was assumed. The notion that the orbits of planets should be a perfect circle was widely accepted, and no scholar would be able to prove otherwise to the people, even Kepler. Thus, it took a long period of time to describe planetary motion in our solar system. The famous three empirical laws of planetary motion describe the motion of the solar system:

- First Law, the law of ellipses: The orbit of each planet is an ellipse with the sun at one focus. The shapes of the planets' orbits are ellipses.
- Second Law, the law of areas: The radius vector to a planet sweeps out equal areas in equal intervals of time. When a planet is closer to the sun, it revolves faster, and, on the other hand, when a planet is farther away from the sun, it revolves slower.
- Third Law, the law of harmony: The squares of the sidereal periods of the planets are proportional to the cubes of the semi-major axes (mean radii) of their orbits. Here, the sidereal period is the time it takes the planet to complete one orbit of the sun with respect to the stars.

Thus, Kepler's laws and Newton's laws taken together imply that a force holds a planet in its orbit by continuously changing the planet's velocity so that it follows an elliptical path. The force is directed toward the sun from the planet and is proportional to the product of masses of the sun and the planet. In addition, the force is inversely proportional to the square of the planet-sun separation. This is precisely the form of the gravitational force postulated by Newton. Newton's laws of motion, with a gravitational force used in the second law, imply Kepler's laws, and the rest of the planets obey the same laws of motion as objects on the surface of the Earth.

Conic Sections and Gravitational Orbits

Various shapes of geometric equations were visualized using conic sections by Hypatia (360–415) for the first time in Alexandria, Egypt (Larson & Edwards, 2010). Conic sections are formed when a cone is cut with a plane at various angles. For a more detailed description, visit the website about this in the Suggested Unit Resources section of this unit.

There are various orbits in a gravitational system. The circular orbit is a special case of an *ellipse*. The ellipse can be formed when the plane intersects opposite edges of the cone. In the case of the parabola orbit, the plane is parallel to one edge of the cone. On the other hand, the hyperbola orbit does not intersect opposite edges of the cone, and the plane is not parallel to the edge. Planets in our solar system have elliptical orbits with various eccentricities. The orbital eccentricity (e) determines the shape of orbits. If $e = 0$ ($E < 0$), the orbit is circular. If $0 < e < 1$ ($E < 0$), the orbit is elliptical. If $e = 1$($E = 0$), the orbit is parabolic, and if $e > 1$($E > 0$), the orbit is hyperbolic. Here, E is the total energy.

Some comets have elliptical orbits too; however, other comets have parabolic orbits, and once they pass the sun, they will never come back. In the case of two interacting stars, they show a hyperbolic orbit. Like the parabolic orbit, the hyperbolic one is a one-time encounter.

Geostationary Orbits

If a satellite revolves around the Earth with the same speed of Earth's rotation, which is about 24 hours a day, it is very useful for communication purposes and is located at about 36,000 km above the Earth's surface. This position does not depend on the mass of the satellite.

The centripetal force is equal to the gravitational force, $mv^2 / r = GmM / r^2$, where m is the mass of the satellite, M is the mass of the Earth, and r is the distance from the center of the Earth to the satellite. The speed of the satellite is $v = (GM / r)^{1/2}$. Also, v = distance / time = the circumference of the circular orbit / the orbital period = $2\pi r / T$.

That is, the orbital period is $T = 2\pi r^{3/2} / (GM)^{1/2}$, or $T^2 = 4\pi^2 r^3 / GM$. Notice that this is exactly the same as Kepler's third law. In other words, the harmonic law is a consequence when the centripetal force is in balance with the gravitational force.

T is for about 24 hours: 1 hour equals 60 minutes; 1 minute equals 60 seconds. Therefore, 24 hours are 24 x 3600 = 86400 seconds. The Earth's mass is 5.98×10^{24} kg and $G = 6.67 \times 10^{-11}$N m^2/kg^2. By substituting all the appropriate values to the above equation, you obtain r = 42300 km. The radius of the Earth is about 6400 km, so subtract 6400 from 42300: 42300 – 6400 = 35900 km, which is about 22300 miles.

Energy Conservation and Escape Velocity

For a circular orbit at distance (r) from the center of the Earth ($r = R_E + h$, if h is the altitude of the orbit), the circular speed (v_c) can be found by equating the centripetal force (mv^2 / r) and gravitational (GmM / r^2) force, or $v = (GM / r)^{1/2}$. Also, from the conservation of total energy, total energy is equal to the sum of kinetic energy and potential energy, so $TE = (KE + PE)_{ground} = (KE + PE)_h$ = constant.

By evaluating TE at h = 0 and at maximum height (h), we find that $mv^2 / 2 = mgh$, or $h = v^2 / 2g$. Notice that when $h = R_E$, v = 11.2 km/s, the projectile escapes to $h = \infty$. This critical speed is called the escape speed.

Escape velocity is defined as the minimum velocity an object must have in order to escape the gravitational field of the Earth. To escape the Earth without ever falling back, the object must have greater energy than its gravitational binding energy.

Black Holes and Einstein's Relativity

A *black hole* is the final stage of massive stellar evolution. Direct observation of a black hole is impossible because no light can escape from it. If you get too close to a black hole, the speed you would need to escape from it would exceed the speed of light. Because nothing can travel faster than light, nothing, not even light, can escape from a black hole. The existence of black holes was predicted by Albert Einstein's General Theory of Relativity, which is the best theory to describe what gravity is and how it behaves. In 1915, Einstein published the General Theory of Relativity. It is all about gravity (Nave, n.d.). The curvature of space and time is influenced by gravity, the more massive the object, the more distortion of space and time.

Work, Energy, and Power

Energy is a quantity assigned to one body that indicates the body's ability to change the state of another body. Notice that energy is not a vector; it is a scalar. Heat is a good example of energy. If a hot object is in contact with a cold object, the hot object will warm the cold one. The quantity ($\frac{1}{2} m v^2$), where m is the mass and v is the velocity of the object, is defined as kinetic energy (KE), and its unit is joule (J).

The work done, W, on an object by a constant force F is defined as $W = Fd$, where d is the displacement. For example, you ride an escalator with your shopping bag, which is hanging straight down from your hand. Your hand exerts a force on the shopping bag, and this force does work. When the escalator goes up, the work is positive because the direction of force is equal to the displacement. On the other hand, the work done is negative when the escalator goes down because the force is opposite to the displacement.

The result of work is a change in the kinetic energy of the object: $W = Fd = mad = mv^2 / 2$ with zero initial velocity. When the force is originated from the accelleration, a, gravity, or g, $W = mgd$. The quantity (mgh), where h is the height of an object, is defined as potential energy (PE), and its unit is also joule (J).

KE arose because of the motion of an object, while PE comes from the object's height.

The total mechanical energy, TE = KE + PE, is conserved; ½ mv^2 + mgh = constant if the net work done by external non-conservative forces such as air resistance and friction is 0. TE is always the same value throughout the motion from the beginning to the end.

Power (P) is work done per unit of time. Its unit is J/s = watt (W); $P = W / t = Fd / t = F v$. That is, the average power is proportional to the average speed under the given force. Power is a scalar quantity because work and time are scalars. The unit *W* is originated from James Watt (1736–1819) who developed the steam engine. The familiar unit horsepower is also used. One horsepower is equivalent to 745.7 *W*.

References

Cutnell, J. D., Johnson, K. W., Young, D., & Stadler, S. (2018). *Cutnell & Johnson physics* (11th ed.). Wiley.

Hewitt, P. G. (2015). *Conceptual physics* (12th ed.). Pearson.

Larson, R., & Edwards, B. H. (2010). *Calculus* (9th ed.). Cengage Learning.

Nave, R. (n.d.). *Principle of equivalence*. HyperPhysics. http://hyperphysics.phy-astr.gsu.edu/hbase/relativ/grel.html

Suggested Unit Resources

In order to access the following resources, click the links below.

This link will take you to the webpage mentioned in the unit lesson. It provides more information about conic sections.

Nave, R. (n.d.). *Conic sections*. HyperPhysics. http://hyperphysics.phy-astr.gsu.edu/hbase/math/consec.html

To learn more about the work-energy principle, take a few minutes to explore the webpage below.

Nave, R. (n.d.). *Work, energy, power*. HyperPhysics. http://hyperphysics.phy-astr.gsu.edu/hbase/work.html

Learning Activities (Nongraded)

Nongraded Learning Activities are provided to aid students in their course of study. You do not have to submit them. If you have questions, contact your instructor for further guidance and information.

1. Solve Questions 50–58 under Physics in Biology, Medicine, and Sports in Chapter 5 of your eTextbook.
2. Solve Questions 70–78 under Physics in Biology, Medicine, and Sports in Chapter 6 of your eTextbook.